PVGO: THE ECONOMICS OF THE PRESENT VALUE OF THE CORPORATION'S GROWTH OPPORTUNITIES

Peter Harding – April 2009

Introduction

For most analysts, it is easy to accept that the present value of all the dividends the corporation will pay over its infinite future life will provide the most obvious estimate of its value, and thus a reasonable predictor of its equity market capitalisation, and thereafter of its stock price. In the case where the dividend may be expected to grow at a constant rate through time, the equity market capitalisation of the corporation may be equated with the dividend it will pay in one year's time divided by the difference between the equity cost of capital and the said growth rate:

$$Q_0 = D_0 \sum ((1+g)/(1+r))^t = D_1/(r-g) \qquad \ldots(1)$$

where the summation runs over the range, t=1 through infinity. This is the well-known Gordon Growth Model, but it will be referred to herein as the 'dividend model'. Q_0 is the equity market value of the corporation immediately after it has paid a dividend of D_0 at the end of period zero: $D_1 (= D_0(1+g))$ is the dividend to be paid at the end of period one, while r and g are the equity cost of capital and the growth rate respectively. For tax reasons it is convenient to assume that the dividend cash is distributed via a sequence of annual stock repurchases. In this way the corporation avoids imposing an income tax liability on those of its stockholders liable to this impost: there is no disadvantage to its other stockholders, who will benefit from an enhanced period by period capital gain, other things being equal. This point was discussed at length in the present author's two papers on the corporation's optimal payout policy (2008) – OPP1 and OPP2 for short in what follows.

A further insight into the valuation of the corporation may be obtained by considering the earnings it will generate in period one (that is, based on its capital employed as at the end of period zero) and the surplus profits it will generate on all the future investments it may be expected to make. Here 'surplus profits' refers to the excess return to be earned on new capital spending over and above the amount necessary fairly to reward the capital – equity and debt - invested therein:

$$Q_0 = E_1/r + PVGO \qquad \ldots(2)$$

where the first term on the right represents the capitalised value of the corporation's equity earnings in period one, and PVGO refers to the net present value of its growth opportunities: equation 2, which is quoted in Brealey, Myers and Allen (2006), will be referred to subsequently as the 'PVGO model'. In the case of equilibrium growth, each new investment year by year generates a perpetual flow of surplus profit for the cohort of stockholders as of the beginning of period one. To reiterate, this profit is over and above the cash needed to service any new equity investment and new debt financing undertaken.

The purpose of the present paper is to demonstrate the nature and origin of the corporation's surplus profits, and to illustrate, in a series of graphs, how such profits flows discount back to the corporation's equity market value. The analysis is based on the model of the corporation developed in OPP1, and without further explanation, Table L1-1 overleaf is extracted from the earlier Table I-1 (page 12 therein).

OPP1 maximised market value added (MVA) by first minimising the cost of capital, and then investing up to that amount each period such that the marginal efficiency of capital expenditure (MEC) falls to the weighted average cost of capital (WACC). This decision is illustrated in Figure L1-2 which appears in a later section of this study. The diagram is adapted from OPP1 (Figure I-2) by adding a horizontal line at the level of the BSWCC: this is the balance sheet weighted cost of capital. The WACC in the original figure is calculated by weighting the equity cost of capital and the interest rate on the corporation's debt by the market values of the equity and of the debt respectively (the latter being always assumed equal to its balance sheet value).

It may be noted that there is a striking contrast in the table between the period one payouts of the corporations growing at 1% and 8%, and their respective equity market capitalisations. This paper will demonstrate that it is the present value of the enhanced growth opportunities of the faster growing of these corporations that compensates for its relatively meagre yield.

TABLE L1-1
P&L ACCOUNTS FOR PERIOD ONE (DEBT/ENTERPRISE VALUE = 25%)

	GROWTH - PER CENT PER ANNUM		
	1	6	8
BALANCE SHEET (BEGINNING PERIOD) - $M			
CAPITAL EMPLOYED	100.000	100.000	100.000
FINANCING			
EQUITY	72.754	69.359	60.731
DEBT	27.246	30.641	39.269
PROFIT AND LOSS ACCOUNT - $M			
OPERATING PROFIT	13.964	13.964	13.964
INTEREST	2.452	2.758	3.534
PROFIT BEFORE TAX	11.511	11.206	10.429
CORPORATION TAX	3.223	3.138	2.920
NET PROFIT	8.288	8.068	7.509
RETAINED PROFIT	0.728	4.162	4.859
PAYOUT	7.561	3.907	2.651
ACCOUNTING RATIOS - %			
ROCE	10.054	10.054	10.054
RETURN ON EQUITY - NET	11.392	11.633	12.365
PAYOUT	91.222	48.420	35.299
INTEREST COVER	569.449	506.356	395.100
STOCK MARKET STATISTICS			
EQUITY CAPITALISATION - $M	81.737	91.922	117.806
ENTERPRISE VALUE - $M	108.983	122.562	157.075
MVA - $M	8.983	22.562	57.075
P-E RATIO	9.862	11.393	15.688
YIELD (DISBURSEMENT) - %	9.250	4.250	2.250
OVERALL TAX RATE - %	22.935	20.484	15.760
PEG	9.862	1.899	1.961
MEC FUNCTION			
INTERCEPT (*100)	10.800	10.800	10.800
GRADIENT (*100)	1.493	0.249	0.187

Further Definition of the Problem

Table L1-2 overleaf is derived from Table L1-1 using the alternative formula for the corporation's equity market capitalisation above (equation 2). The analysis will focus on the case of 6% growth as in OPP1, and data for the 1% and 8% cases are included in the table by way of comparatives.

The table begins with the equity market capitalisation and earnings (net profit) from Table L1-1 in each case. Capitalising earnings by dividing by the equity cost of

TABLE L1-2
PRESENT VALUE OF GROWTH OPPORTUNITIES ($M.)

	RATE OF GROWTH (% PER ANNUM)		
	1	6	8
EQUITY MARKET CAPITALISATION	81.737	91.922	117.806
CURRENT EARNINGS	8.288	8.068	7.509
PV OF CURRENT EARNINGS: A	80.859	78.712	73.259
PV OF GROWTH OPPORTUNITIES: B	0.878	13.210	44.547
RECONCILIATION:			
BALANCE SHEET EQUITY: C	72.754	69.359	60.731
PV OF CURRENT SURPLUS PROFITS: A-C	8.105	9.352	12.528
MARKET VALUE ADDED: B+(A-C)	8.983	22.562	57.075

capital – 10.25% in all cases - results in the third row, ‘present value of current earnings’ (PVCE – labelled A). These data may then be deducted from the equity market capitalisation in each case to give the PVGO (labelled B), as indicated by equation 2. In this sense, the PVGO is derived simply as a residual. The force of a higher growth rate is immediately evident. The equity market capitalisation of the corporation growing at 8% exceeds that of the corporation growing at 1% by some $36.069m.: the PVGO of the former exceeds that of the latter by no less than $43.669m. Both these cases have initial capital employed of $100m. and period one net operating profit after tax (NOPAT) of $10.054m. (this latter figure may be compared with the first of the accounting ratios in the table).

The next row (labelled C) shows the balance sheet equity in each case as extracted from Table L1-1 again. By deducting balance sheet equity from the PVCE (indicated by A-C), the present value of current surplus profits (PVCSP) is obtained. It may then be observed that the sum of this item and the PVGO results in the MVA (that is, B+(A-C)). This proves the figures for market value added set out in Table L1-1 earlier. The purchaser of the corporation’s stock may then be seen as acquiring an asset with three components: a share in its capital employed (labelled C), the same share of the surplus profits to be generated by these assets in perpetuity (A-C), and the same share again in the surplus profits on all the investments the corporation will make in the future (B). In the case of 6% growth, the value of the balance sheet equity is $69.359m., the PVCSP is $9.352m. and the PVGO is $13.210m.: together these three items sum to the corporation's equity market capitalisation of $91.922m. Maximising MVA effectively means maximising PVGO as this is the actionable component of the corporation's value. For the purposes of the analysis, it is assumed that the existing assets and the flow of income from them are fixed for all time. Of course, this will generally not be the case: normally a substantial amount of management time will be devoted to improving the efficiency and profitability of its existing operations.

It is the aim of this study to make sense of these amounts diagrammatically, including the shape of the 'cash flow' implicit in the formula for the equity market capitalisation of the corporation given by equation 2 above. The analysis will involve recourse to the study by Miller and Modigliani (1961), in which they use home-made dividends to illustrate the insensitivity of the corporation's equity market capitalisation to its payout ratio. In the following section, the profit to be generated in perpetuity by the period one capital expenditure of $6m. is illustrated. Later sections graph the build-up of the equity market value of the corporation in terms of the variables set out in Table L1-2, and in terms of the dividends it may be expected to distribute.

MEC: The Corporation's Demand for Investment Funds

OPP1 offered a brief outline of the derivation of the marginal efficiency of capital expenditure function (MEC). Here it is possible to be somewhat more expansive. By way of illustration, Table L1-3 overleaf lays out a possible set of potential investment projects identified by the corporation growing at 6% for implementation at the end of period one.

The table shows five possible projects, labelled A through E, including initial capital cost, annual net operating profit after (corporation) tax (NOPAT), and the rate of return for each. The rate of return is the ratio of the NOPAT to the capital cost in each case, as it is assumed that each is in the simple form of a single cash outlay followed be an infinite constant flow of cash receipts: insofar as the assets will require

TABLE L1-3
CAPITAL EXPENDITURE BUDGETING - PERIOD ONE

PROJECT	$M.	CAPITAL COST	ANNUAL NOPAT	PROJECT ROR (%)	CUMULATIVE: CAPITAL COST	ANNUAL NOPAT	ROCE (%)
A		1.000	0.107	10.676	1.000	0.107	10.676
B		2.000	0.206	10.303	3.000	0.313	10.427
C		2.000	0.196	9.805	5.000	0.509	10.178
D		1.000	0.094	9.432	6.000	0.603	10.054
E		1.000	0.092	9.183	7.000	0.695	9.929

NOTE: CUT-OFF RATE OF WACC - 9.3075%

some maintenance expenditure to sustain their earning power over such a long horizon, it is assumed that such expense has been allowed for in arriving at the NOPAT. If undertaken, each investment will be made at the end of period one, with the first of the resulting cash inflows commencing at the end of period two.

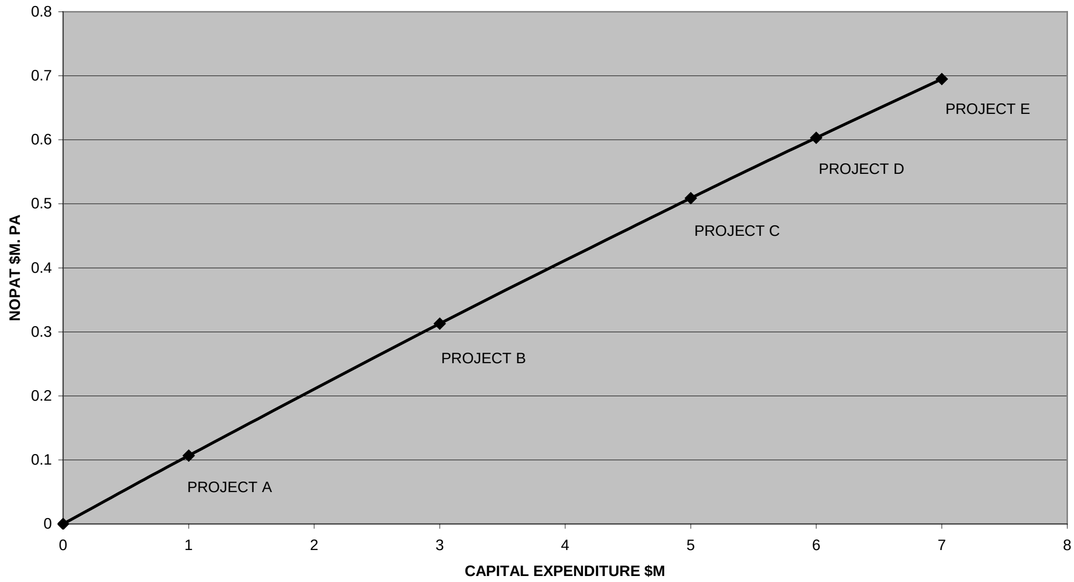
As may be noted, the five projects have been ranked in rate of return order. On the right of the table, then, it is possible to list their cumulative capital cost starting with the most profitable, the cumulative NOPAT, and the average rate of return on the first several projects. Figure L1-1 overleaf plots the cumulative data.

In fact, the points have been constructed so as to lie on a quadratic, and as part of the modelling process, this relationship has been inked in:

$$\text{NOPAT} = 0.108X_1 - 0.00124375X_1^2 \qquad \ldots(3)$$

where X_1 represents the period one level of capital expenditure in millions of dollars. Somewhat unrealistically, this simplification implies that there is only one possible project, but that it may be scaled to any size, with the rate of return falling linearly as it is scaled up (and with the limiting rate of return is set at a maximum of 10.8% net).

FIGURE L1-1



The marginal efficiency of capital expenditure is then obtained as the derivative of this quadratic:

$$\text{MEC} = 0.108 - 0.0024875\text{X}_1 \qquad \ldots(4)$$

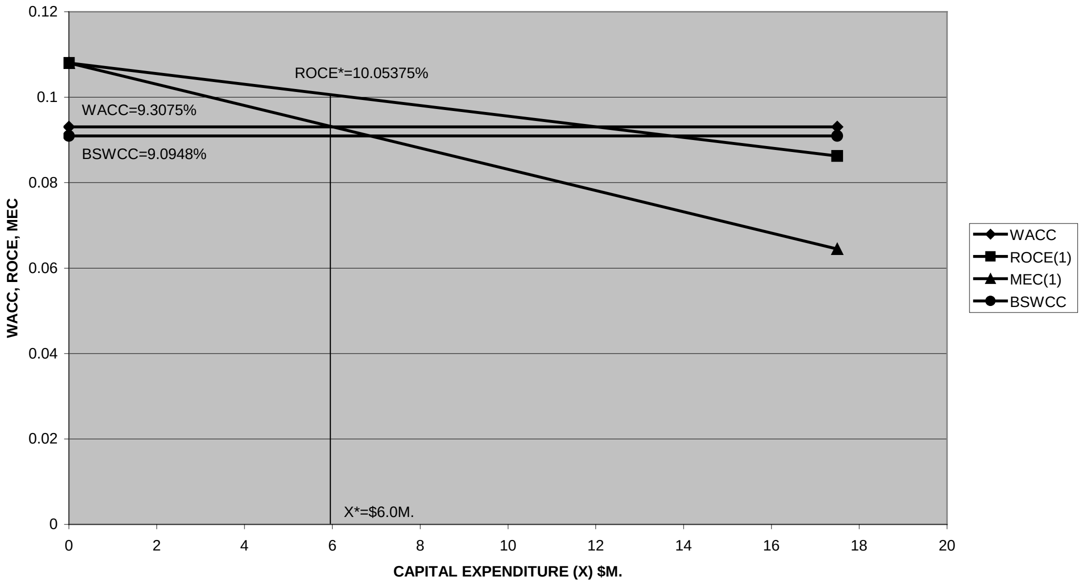
Equation 4 presents as a straight line with intercept of 0.108 and negative slope of 0.0024875: these parameters are as detailed at the foot of the 6% growth column in Table L1-1. The return on capital employed - obtained by dividing through the NOPAT by X_1 - similarly has an intercept of 0.108, but its gradient is half that of the MEC:

$$\text{ROCE} = 0.108 - 0.00124375\text{X}_1 \qquad \ldots(5)$$

The ROCE figures in Table L1-3 fall on this line. These relationships are depicted in Figure L1-2 overleaf: both the MEC and the ROCE may be thought of as ratios akin to a rate of interest, whereas the NOPAT in Figure L1-1 is measured in millions of dollars.

Figures L1-1 and L1-2 follow the analysis set out in Chapter 14 of Gordon (1962), in particular his Figure 14.2 on page 209.

FIGURE L1-2



It was established in Figure I-1 of OPP1 that the corporation's optimal gearing ratio is 25%, and that this would result in a WACC of 9.3075%. The WACC has been drawn in at this level in Figure L1-2. The optimal level of capital expenditure in period one was shown to be determined where the WACC cuts the MEC. The MEC thus acts as the corporation's demand curve for investment funds: in this case the optimal value of X_1 works out at $6m. Projects A through D in Table L1-3 have rates of return in excess of the WACC, but Project E fails the test. It may further be seen from the table that these four projects generate an addition to period two income of $0.603225m.: this extra income will henceforward be received in perpetuity. By raising the perpendicular at the optimal level of capital expenditure, it may be calculated that the area enclosed by this construct, the axes and the MEC equates to this additional annual NOPAT (=6*(0.108+0.093075)/2): in making this calculation, it is necessary to rely on the linearity of the MEC. Of course, the same result is obtained as the product of the optimal level of period one capital expenditure and the ROCE – 10.05375% as indicated in the figure. The ROCE is shown as the first of the accounting ratios in Table L1-1: it also appears as the cumulative rate of return on projects A through D in Table L1-3.

Keynes (1936) devoted Chapter 11 of his General Theory to the 'marginal efficiency of capital', and the concept plays a significant role in his subsequent discussion of the functioning of the economy. Of course, his analysis is aimed at the aggregate level, and he obtains his ‘investment demand schedule’ by adding together the MECs of all the firms operating in the economy. In due course, it is appropriate to envisage a steady rightward shift in the corporation’s MEC over time, such that its optimal level of demand for investment funds grows at a constant rate from period to period. In technical terms, this is achieved by reducing the gradient of the MEC by successively multiplying by the factor, 1/(1+g), where g is, as before, the growth rate.

The Rate of Surplus Profit

Figure L1-2 is a repeat of Figure I-2 of OPP1, except that it has been modified to show the balance sheet weighted cost of capital (BSWCC) as well as the WACC (which, as noted previously, is market capitalisation weighted). The balance sheet weighted cost of capital for the case of 6% growth may be computed using the amounts of equity and debt from the balance sheet shown in Table L1-1:

$$\text{BSWCC} = 69.359 * 0.1025 + 30.641 * 0.0648 = 9.0948343\% \qquad \ldots(6)$$

This is the figure shown in the diagram, based on the ECC of 10.25% and a net (of corporation tax) rate of interest of 6.48% (=9*(1-0.28)). As noted, the corporation invests $6m. in period one, financed by equity of $4.162m. and debt of $1.838m. The former figure appears in the table as retained profit, and represents some 69.359% of the amount to be invested: this proportion is derived from the balance sheet value of the equity in Table L1-1.

The difference between the return on capital employed and the balance sheet weighted cost of capital represents the rate of surplus profit earned by the corporation:

$$\text{ROCE} - \text{BSWCC} = 10.05375 - 9.0948343 = 0.9589157\% \qquad \ldots(7)$$

This is the surplus that accrues to the stockholders after both the equity and the debt have been rewarded at their respective market rates. Applying the surplus return to the period one capital expenditure of $6m. indicates a financial surplus of:

$$X_1(\text{ROCE} - \text{BSWCC}) = 6.0 * 0.009589157 = \$0.057535\text{m.} \qquad \ldots(8)$$

to be generated in perpetuity commencing at the end of period two. Discounting at 10.25%, the equity cost of capital, results in a net present value of \$0.561317m. at the end of period one when the investment is made. For future reference, it may be noted that this figure represents 6% of the figure for the present value of current surplus profits (\$9.352m.) in Table L1-2 above.

As a footnote to this section, it may be noted that a higher level of profitability for the corporation may be modelled by assuming an increased intercept for the MEC in Figure L1-2. If the intercept were 0.144 (this represents a net of tax rate of return of 14.4% on a vanishingly small investment outlay) and the gradient were (negative) 0.0084875, the optimal level of capital expenditure would again be \$6m. The ROCE would then work out at 11.85375%: the intercept of the ROCE function is the same as for the MEC and the gradient is halved as before. Assuming 6% growth again, the application of equation I-17 of OPP1 then results in an enterprise value of \$176.984m.:

$$V_0 = A_0(\text{ROCE} - g)/(\text{WACC} - g) = 100.0 * 0.0585375/0.0425 \qquad \ldots(9)$$

where V_0 and A_0 (=100) represent enterprise value and capital employed respectively at the outset of period one. One quarter of this amount, \$44.496m., will be the corporation's outstanding debt, giving a balance sheet gearing ratio of 44.496%. Using the factor costs shown in equation 6 above, a BSWCC of 8.5725% may be confirmed, which, by analogy with equation 7, implies a surplus rate of return of 3.281% (=11.854-8.573): on the period one investment of \$6m.this equates to a perpetuity of \$0.197m. These figures compare with figures of only 0.959% and \$0.057535m. respectively for the 6% growth case in Table L1-1.

The Present Value of the Growth Opportunities

The steady growth of the corporation is assumed to be generated by an underlying period to period shift in its MEC: the gradient of the MEC falls yearly by multiplying by the factor 1/(1+g), thus increasing the demand for investment funds by g per cent per annum. At the end of period two, the corporation's optimal level of investment will thus increase by 6% to \$6.36m. At the previously computed rate of excess return, this will generate a perpetual flow of profits of \$0.060987m. (=1.06*0.057535) commencing at the end of period three. This has a present value of \$0.594995m. at the end of period two when this investment is made: this is 6% greater than the present value of the of the flow generated by the period one investment, and the pattern becomes clear. The process thus sets up a sequence of amounts – the net present values of successive investments - to be valued: the sequence starts at \$0.561317m. at the end of period one and growth takes place through infinity at 6% per annum. Discounting at the equity cost of capital gives:

$$\begin{aligned} PVGO &= X_1(ROCE - BSWCC)/[r(1+r)] + \\ &\qquad (1+g)X_1(ROCE - BSWCC)/[r(1+r)^2] + ... \\ &= X_1(ROCE - BSWCC)/[r(r - g)] \qquad \ldots(10) \end{aligned}$$

where the dividend discount formula itself (equation 1 above) provides the model for the summation process. Substituting in the value for the period one investment computed above gives:

$$PVGO = 0.561317/(0.1025 - 0.06) = \$13.210m. \qquad \ldots(11)$$

This is the figure for the PVGO in the case of 6% growth in Table L1-2.

Looking Backwards: Period One Surplus Profit

It is possible to look backwards, and ask how much surplus profit the \$100.0m. of assets owned by the corporation at the end of period zero will generate in period one.

It has already been calculated that the \$6.0m. invested at the end of period one will generate an excess profit of \$0.057535m. in perpetuity starting at the end of period two. The period zero capital expenditure would be smaller by the factor 0.949336 (=1/1.06), and the excess profit correspondingly smaller. Continuing this retro-gradation, it is then evident that the perpetuity formula may be applied to give a figure for the surplus profits generated in period one of \$0.958917m. (=0.057535/0.06). This figure for the excess profit may be extracted from the accounts in Table L1-1, as shown in Table L1-4 overleaf.

The first entry is the net profit (earnings) for period one for the case of 6% growth from Table L1-1. Adding the net interest (gross interest less the value of the tax

TABLE LI-4
MANAGEMENT ACCOUNTS - PERIOD ONE (6% GROWTH)

		$M.	$M.
NET PROFIT			8.068
INTEREST		2.758	
	LESS TAX SHIELD AT 28%	0.772	1.986
NOPAT			10.054
COST OF CAPITAL			
	EQUITY: $69.359 @ 10.25%	7.109	
	DEBT: $30.641 @ 6.48%	1.986	9.095
	SURPLUS PROFIT		0.959

shield) results in NOPAT of $10.054m.: this corresponds to the ROCE, shown as the first of the accounting ratios in Table L1-1. Against the NOPAT, it is necessary to charge the cost of capital (BSWCC), some $9.095m., given capital employed of $100.0m. This results in a figure of $0.959m., which was the figure encountered in the previous paragraph. Capitalising this amount at the ECC, 10.25%, results in a value of $9.352m., which appears as PV of current surplus profits in Table L1-2 above.

As the whole of the excess profit accrues to the stockholders, it is meaningful to relate it to the accumulated equity investment at it stands at the beginning of period one – some $69.359m. The ratio represents an excess return of 1.383%: together with the equity cost of capital, 10.25% charged in Table L1-4, this indicates a return on equity (ROE) of 11.633%. This proves the second of the accounting ratios given in Table L1-1.

This analysis demonstrates the tension that exists in the constant growth model between the WACC and the BSWCC. The former is critical in determining the corporation’s optimal level of investment each period. The latter is necessary to determine period by period in the management accounts the extent to which the corporation is creating value for its stockholders. Equation 6 above calculated the

BSWCC for the case of 6% growth to be 9.095%: the corresponding figures for 1% and 8% growth are 9.223% and 8.770% respectively. Return on equity is thus seen to be an increasing function of growth as shown in in Table L1-1 above and Figure II-4 of OPP2.

Returning to the more profitable corporation introduced in the paragraph at the end of the last section but one (MEC intercept of 0.144, etc.), it is possible to calculate that it will manifest a ROE of 16.162%. At the end of period one, it will retain \$3.330m. (=6(1-0.44496)) as the equity contribution towards its \$6m. capital expenditure programme. The applicable surplus profit \$0.197m. calculated above represents 5.912% of this amount, which, added to the ECC of 10.25%, results in the ROE.

'Inside the P/E Ratio': A Digression on Leibowitz and Kogelman (1990)

The definition of the ROE in the last paragraph of the previous section permits the translation:

$$X_1(\mathrm{ROCE} - \mathrm{BSWCC}) = g(A_0 - B_0)(\mathrm{ROE} - \mathrm{ECC}) = 0.057535 \qquad \ldots(12)$$

where A_0 and B_0 are respectively the corporation's capital employed and outstanding debt at the end of period zero, as defined in OPP1. Multiplying the difference between them – that is, the balance sheet equity - by the growth rate results in the new equity to be invested at the end of period one: this, it was noted earlier, is the figure for retained profits of \$4.162m. as shown in Table L1-1. Combining equations 2, 4 and 6 gives:

$$Q_0 = E_1/r + g(A_0 - B_0)(\mathrm{ROE} - r)/[r(r - g)] \qquad \ldots(13)$$

where r has been substituted for the ECC as usual. Since:

$$E_1 = (A_0 - B_0) * \mathrm{ROE} \qquad \ldots(14)$$

the equity market capitalisation may restated as:

$$Q_0 = E_1/r + gE_1(\mathrm{ROE} - r)/[\mathrm{ROE} * r(r - g)] \qquad \ldots(15)$$

whence, dividing through by period one earnings, the prospective price-earnings ratio is obtained as:

$$Q_0/E_1 = 1/r + g(ROE - r)/[ROE * r(r - g)] \quad \text{...(16)}$$

This Leibowitz and Kogelman (1990) translate into:

$$P/E = 1/r + G * FF \quad \text{...(17)}$$

where 1/r, the inverse of the ECC, is the base level of the price-earnings ratio and:

$$G = g/(r - g) \quad \text{...(18)}$$

they term the 'growth factor', and:

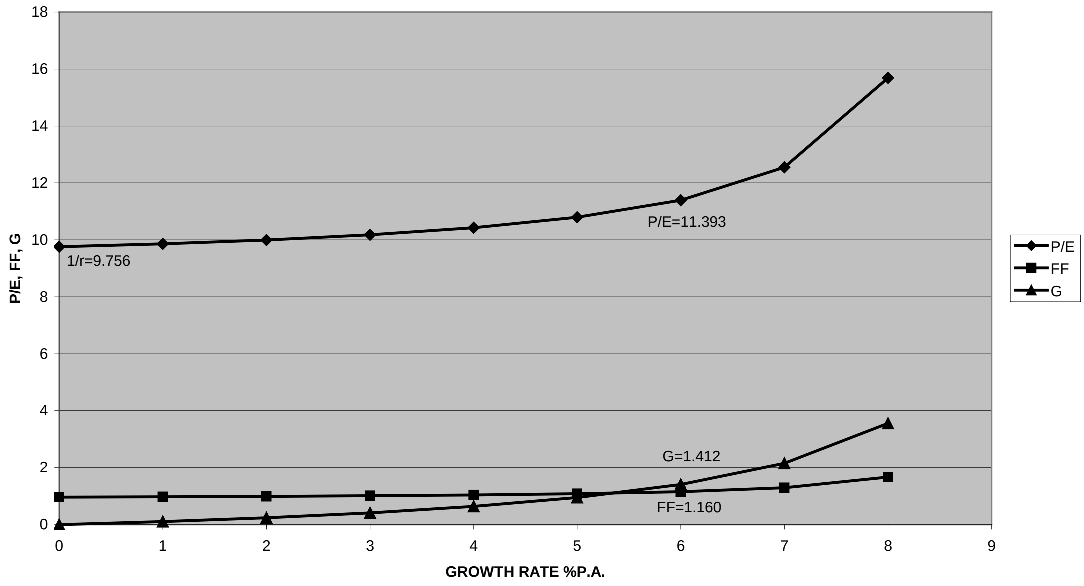
$$FF = (ROE - r)/ROE * r = 1/ECC - 1/ROE \quad \text{...(19)}$$

the 'franchise factor'. These factors provide a route towards a deeper understanding the escalation in the P/E ratio with the growth rate. Their formula is certainly an improvement on equation II-4 of OPP2, depending as it does on only three variables for which estimates may be developed fairly readily. However, these are arranged in such a complex manner that it defies belief that analysts can make intuitive judgments about the correct P/E valuations for stocks, especially when, in practice, earnings fluctuate so randomly through the economic cycle.

The P/E and the factors that drive it are graphed together in Figure L1-3 overleaf.

It is clear that g=0 eliminates the second term in the formula and reduces the P/E ratio to its base level of 9.756 (=1/0.1025), as shown in the figure: this was the ratio listed for the case of the zero growth corporation in Table I-1 (page 12) in OPP1. The table indicates a ROE of 11.376% for this company, which results in a franchise factor of 0.966. As may be readily deduced, the growth factor is an increasing function of the growth rate, and this drives a rise in the equity market value of the corporation: it summarises the scale of the investments the corporation will make in the future relative to its initial (period one) capital employed. But the growth rate also drives an increase in the value of the franchise factor. The rising value of the equity with growth supports an increasing debt burden: for instance, the debt ratio rises from 27.248% of capital employed in the case of 1% growth (Table L1-1), to 39.269% for growth at 8%. This is reflected in a falling level of the BSWCC in Figure L1-2,

FIGURE L1-3



which then increases the respective ROEs from 11.392% to 12.265%, with the corresponding franchise factors increasing from 0.978 to 1.669. There is thus both a direct contribution to the P/E ratio from the rate of growth, and an indirect effect through the ROE and thus the franchise factor.

In the case of 6% growth, the franchise factor is 1.160 and the growth factor works out at 1.412. Substituting these values into the Leibowitz and Kogelman formula returns a P/E of 11.393, as shown in Table L1-1. These data are included on the face of Figure L1-3.

The earlier more profitable corporation (with the elevated MEC intercept) will have the same base level P/E ratio and the same growth factor for g=0.06, but the franchise factor will now work out at 3.569 (=1/0.1025-1/0.16162): in this case then, the P/E ratio becomes 14.794, a premium of almost 30%. It will be recalled that the ROCE in this case 11.854% compared with the figure of 10.054% shown for the 6% growth case in Table L1-1. This is not the place to speculate on the extent to which the higher rate of return of the more profitable corporation might be sustainable in a fully competitive economic environment.

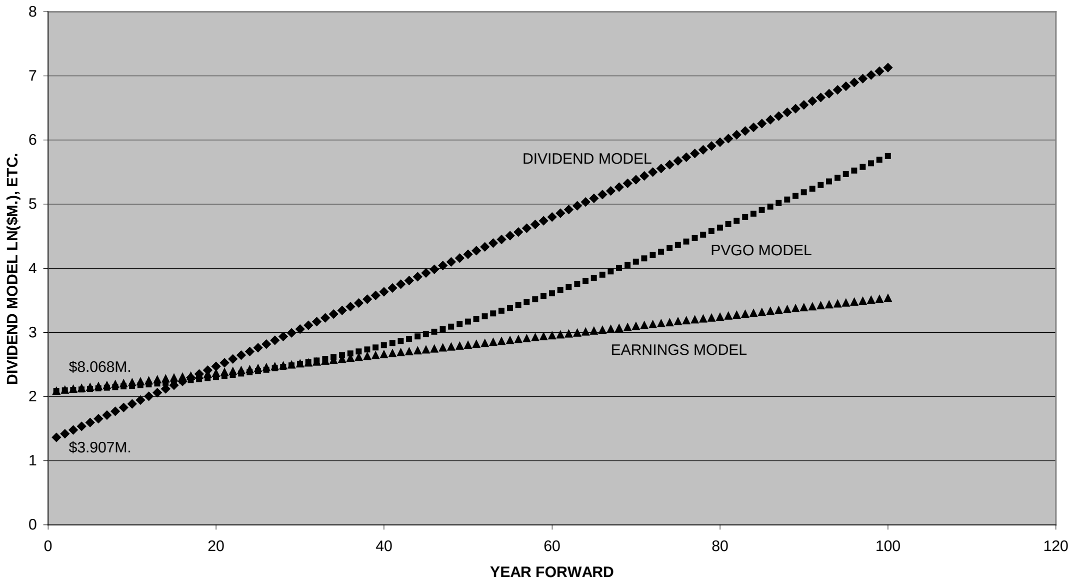
Home-made Dividends

The dividend flow associated with the constant growth formula in equation 1 above is relatively straightforward. The corporation pays out what it can afford after prioritising its capital expenditure requirements each period: it is self-financing in terms of equity – of course, borrowings increase in proportion from period to period as well – so that earnings and dividends grow at the same rate over time and the payout ratio remains constant. But this then raises the question of the nature of the cash flow implicit in equation 2 – the PVGO model: as noted, this has the same present value as the dividend model, which is, in turn, equal to the equity market capitalisation of the corporation.

In order to examine this question, it is helpful to replicate the figure (Figure 1 – p.423) devised by MM (1961) to demonstrate their dividend irrelevancy proposition. As usual, the analysis will be based on the case of 6% growth in Table L1-1. It will also make the argument simpler if personal taxes are ignored, and the corporation pays a cash dividend period by period: the same result may be obtained if the corporation makes its distribution through the repurchase route – in which case, it would be appropriate to disregard personal taxes anyway – but the argument would be at risk of becoming obscured by the more complicated arithmetic. Thus in Figure L1-4 overleaf the diamonds represent year by year the amounts paid out by the 6% corporation in Table L1-1 above. In fact, the vertical axis is measured in terms of the natural logarithm of the dividends, so that the period one payout of $3.907m. – the first diamond - has been entered explicitly: as growth takes place at a constant 6% rate, the logarithm of the series presents as a straight line. Equation 1 thus encapsulates what has been termed the 'dividend model' of the equity market value of the corporation.

MM contrast the dividend model with what may then be called the 'earnings model': according to this latter system of cash flows, the corporation always pays a dividend equal to its total earnings. Thus it is necessary for it to sell sufficient new shares period by period to finance the equity portion of its optimal investment plan. Its capital expenditures and its total earnings therefore follow the same path as in the 'dividend model': the earnings series is not shown in the figure, but may be imagined as a series starting at the period one earnings figure of $8.068m. in Table L1-1 – the

FIGURE L1-4



positioning of this value is again indicated explicitly - and proceeding upwards to the right as a straight line parallel to the diamonds (that is, with the same 6% growth rate). The triangles then plot the dividends to be received year by year by the cohort of stockholders as constituted at the commencement of period one: it is their interest in the corporation, as opposed to the interests of any subsequent stockholders, that is captured by the market capitalisation as at the beginning of period one. As the corporation is selling new shares to outsiders each period, the interest of the original cohort is continually diluted: as a result, their income increases at the lower rate of 1.473%, which is the difference between the ECC (10.25%) and the earnings yield – 8.777% (=100*8.068/91.922). This yield now coincides with the dividend yield, of course, so that the earnings model thus enables the corporation to claim credit for a higher dividend yield than its counterpart following the dividend model. This process of dilution was described in OPP1, but it is worth repeating the exercise in the present lemma to complete the explanation of the lower growth rate.

As in OPP1, it will be assumed that the corporation has 100m. shares in issue at the beginning of period one. The stock will thus trade at 91.922c. initially – this figure is derived from the market capitalisation shown in Table L1-1 - but the price will have risen by 10.25% to 101.344c. cum dividend by period end. Given the 100% payout ratio, the corporation then pays out a dividend of 8.068c. per share, which will reduce the stock price to 93.276c. ex dividend: this stock price is 1.473% greater than the ex dividend price one year earlier. The corporation now needs to raise $4.162m. (the figure for retained profit in Table L1-1 above) to finance the equity portion of its optimal period one capital expenditure programme of $6m. Assuming new shares may be sold into the market at the ex dividend market price – a rights issue at a discounted price would again unduly complicate the arithmetic - the result is an increase in the number of shares in issue of 4.462m. (=4.162/0.93276). As a result of the investment of $6m. at the end of period one, the period two earnings of the corporation will be 6% greater at $8.552m. When this is paid out as a dividend, the amount to be received by the period one cohort of stockholders will be $8.187m. (=8.552*100/104.462), due to the dilution of their interest as a result of the equity issue. Thus the dividend received by the period one cohort at the end of period two - 8.187c. per share - is then seen to be only 1.473% greater than the 8.068c. received at the end of period one. Again, together with the earnings/dividend yield of 8.777%, the stockholders are seen to enjoy a return equal to the equity cost of capital – 10.25%.

The process of the dilution of the interest of the period one cohort of stockholders continues ad infinitum, so that the sequence of dividends may be represented in Figure L1-4 as a straight line – the triangles - with an initial value of $8.068m. as indicated, and a constant slope when graphed in logarithmic terms, reflecting growth of 1.473%. This demonstrates the irrelevancy of the payout ratio to the value of the corporation, and the argument may be extended to encompass any intermediate policy. If the corporation decides on a 75% payout ratio, for instance, the cash flow would be represented by a straight line with a period one dividend of $6.051m.: this would give a dividend yield of 6.583%, so that the slope of the resulting straight line would reflect a rate of increase in the dividend of 3.667% per annum. At a discount rate of 10.25%, a series with an initial dividend of $6.051m. that is growing at 3.667% will have a present value of $91.922m., the same as the 6% growth and 4.25% yield case in Table L1-1. (For the reasons stated in OPP1, the present author believes the MM analysis devalues excessively and unnecessarily the payout ratio associated with the ‘dividend model’ – 48.420% as shown in Table L1-1).

While it has been convenient to think in terms of an initial cohort of stockholders that stays with the corporation over the long term, this is obviously an over-simplification. In particular, stockholders may exchange any of the cash flows discussed above for any other by buying or selling in the market. Rather than to demonstrate this process by considering an initial cohort that is invested in a corporation that pursues the dividend model, and that wants to harvest its interest according to the earnings model, or vice versa, it is now more practical, and indeed more pertinent, to build on the previous analysis by examining the process of converting the cash flow of the earnings model into that of the PVGO model. Thus, at the beginning of period two, the corporation has 104.462m. shares in issue, of which 100m. remain in the hands of the period one cohort. A similar process of dilution takes place at the end of period two. During period two the stock price will rise by 10.25% from 93.276c. to 102.837c. cum dividend. As calculated earlier, the corporation will earn $8.552m. in period two which works out at a dividend of 8.187c. per share (=8.552/1.04462): the price of the stock will fall to 94.650c. on going ex dividend. The corporation now needs to raise new equity of $4.412m. (=4.162*1.06) at the end of period two: at the ex dividend price, this requires the sale of 4.661m. new shares, bringing the total in issue to 109.123m. Period three earnings will rise a further 6% to $9.065m., of which only some $8.307m. (=9.065*100/109.123) will be attributable to the period one cohort. This amounts to 8.307c. per share, and again, their income rises at the rate of only 1.473% (the shares still trade on a yield of 8.777%, of course).

The cash flow underlying the PVGO model envisages that the period one cohort will pocket in full the dividend declared in period one, but that it will decide to buy some of the shares sold in the equity issue at the end of period two. They augment their spending money only by an amount equal to the surplus profit to be earned in perpetuity by the investment of $6m. at the end of period one: this was previously calculated to be $0.057535m. Of the 4.661m. new shares to be sold by the corporation, the cohort buys some 0.064m., costing $0.061m. at the ex dividend price of 94.650c. per share. The cohort then enjoys period two spending money of some $8.126m. (=8.187-0.061). This is equivalent to the period one dividend of $8.068m. plus the surplus profit of $0.057535m.: their home-made dividend at the end of period two thus manifests an increase of only 0.713% over the previous year, less than the 1.473% growth of the earnings model. At the beginning of period three, the period one cohort owns 100.064m. shares with a value, at 94.650c.xd. each, of some \$94.711m. A year earlier the value of their holding in the corporation amounted to \$93.276m. (=100*0.93276): this represents an increase of 1.538%, which, together with a prospective 'income' yield at that time of 8.712% (=100*8.126/93.276) indicates a total rate of return of 10.25%, the equity cost of capital as usual.

It is necessary to pursue these calculations a step further, and it may additionally be noted that the period one cohort now augments its spending money again in period three by the amount of the excess profit generated in perpetuity by the investment made at the end of period two: the balance of its period three dividend increase is applied to purchasing further shares. In total, the home-made dividend taken at the end of period three becomes \$8.187m. (=8.068+0.057535+1.06*0.057535), by coincidence equal to the period two dividend under the earnings model: this time the increase amounts to 0.751% - compared with 0.713% in period two - and the growth rate of the series may then be assumed to be accelerating over time. The continuation of the series may be seen as the squares in Figure L1-4. The flow of spending money implied by the PVGO model falls below that of the earnings model, shown by the triangles, for about 30 years. The 'curve' in due course becomes asymptotic to a straight line with the same slope as the diamonds - . As the 'initial conditions' – the assets held by the corporation on behalf of the period one cohort - fade into history, a growth rate of 6% establishes itself, both for the growth of the home-made dividends, and the annual increase of the value of their interest in the corporation. But after 100 years, this interest will have been diluted substantially, down to only about one eighth of its original 100%.

Obviously equation 2, the PVGO model, implies a highly contrived cash flow, which no stockholder would ever be expected to engineer as a way of realising the value of her investment in the corporation. Its interest lies in the decomposition of the equity market capitalisation of the corporation which it permits. This is demonstrated in the graph in the following section, which also offers a visual impression of how the sums of the present values of the diamonds and the squares in Figure L1-4 converge on the equity market capitalisation of the corporation. The same is true of the present value of the triangles also (the earnings model), of course, but this series is not included in the discussion beyond this point, having served its purpose as the basis of the derivation of the cash flow implied by the PVGO model. In fact, it is possible to infer from Figure L1-4 that the discounted cash flows of the PVGO model and the earnings model must largely coincide: they share a common initial value, and the present values of both series must be asymptotic to zero. The actual cash flows begin to diverge significantly only after about 40 years, as shown, but by this time into the future the discount factor will have fallen to approximately one fiftieth ($= (1/1.1025)^{40}$), thus overwhelming the growing divergence between the series shown in Figure L1-4.

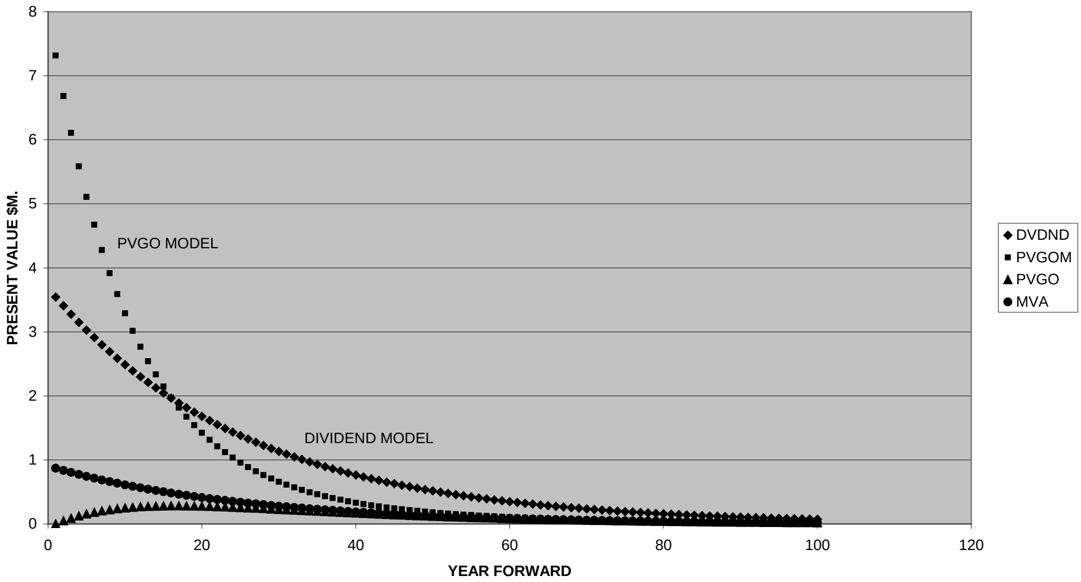
The Alternative Models of Equity Market Capitalisation Compared

In the introduction to this lemma, equation 2 separated equity market capitalisation into the present value of the growth opportunities (PVGO) to be generated by the additional assets to be invested in by the corporation over its infinite future and the present value of the future earnings to be generated by the equity-financed portion of the assets in situ at the beginning of period one (itself subsequently revealed in Table L1-2 to be the sum of two components). Of course, this same equity market capitalisation is also equal to the present value of the dividends the corporation will pay in the future, as stated in equation 1 and shown as the diamonds in Figure L1-4 above - although, for tax reasons, it was assumed that the corporation would take the repurchase route in its distribution policy. The repurchase process does not affect the corporation's economics – the values over time of its capital employed, its earnings and its market capitalisation: but it does affect the number of shares in issue, and thus also the stock price and the reported earnings per share.

Figure L1-5 below shows the build-up of these present values over time – though for practical reasons, the graph has been truncated at 100 years.

The diamonds now represent the present value of the dividend to be paid each year discounted back to the present, the beginning of period one. The first value is the payout of \$3.907m. shown in Table L1-1, discounted for one period at the equity cost of capital, 10.25%. The dividend grows at 6% through to infinity, and the discounted amounts sum to \$91.922m., the equity market capitalisation of the corporation. If the diamonds were to be viewed as a curve proceeding through to infinity, the area under

FIGURE L1-5



the curve could be interpreted as approximating the corporation's equity market capitalisation. The duration of a dividend flow growing at 6% per annum and discounted at 10.25% works out at about 26 years.

This area ($91.922m.) is also equal to the sum of the squares, representing the PVGO model as in Figure L1-4. The initial point on this 'curve' is the figure for net profit (earnings) in Table L1-1, $8.068m., discounted at the ECC. In its turn, the present values of this series are equal year by year to the sum of three other series.

The first of these, shown as the triangles in Figure L1-5, is the addition to present value, year by year, of the excess profits to be generated by the additional investments the corporation makes at the end of period one, and at the end of each period thereafter. The area under this curve adds up to the PVGO, $13.210m., as derived earlier in Table L1-2. The period one value of this curve is zero. As noted previously, the corporation invests $6.0m. at the end of period one, and this generates an excess profit of $0.057535m. per annum at the end of period two and thereafter: the second triangle represents this amount discounted over two periods. Again as noted earlier, at the end of period two the corporation invests $6.36m. (6% more) to generate an additional excess profit through infinity of $0.060987m., commencing at the end of period three. The sum of the extra surplus profits at the end of period three is thus \$0.118522m. (=0.057535+0.060987), and the third triangle represents this amount discounted over three periods. This curve builds up to a maximum present value of \$0.281167m. in year 17: by this time, the additional investments over the intervening years will be generating a surplus profit before discounting of \$1.477m. $(= 0.281167 * 1.1025^{17})$.

The second component, shown in the figure as the sliver between the triangles and the asterisks, represents the surplus profits to be generated in perpetuity by the assets owned by the corporation at the beginning of period one. The period one value of the series is the figure of \$0.959m. derived as the period one excess profit in the management accounts presented as Table L1-4, discounted at the ECC for one period. This same amount is discounted through infinity to give the capital sum of \$ 9.353m. (=0.959/0.1025): this figure was derived earlier from the corporation's accounts in Table L1-2.

The area under the asterisks (this is now taken to include the area under the triangles) sums to the corporation's market value added: this amounts to the figure of \$22.562m. shown in Tables L1-1 and L1-2 earlier. Thus, the PVGO is seen to be one of two components of the MVA, which was adopted in OPP1 as the magnitude to be maximised by the corporation. The ultimate cash flow from the corporation to the stockholders – the yearly dividends or repurchases – is matched by a shadow or quasi cash flow, referred to above as the PVGO model. The component below the asterisks is the present value of a flow starting with a value of \$0.959m.: it grows at 6% per annum, and discounting at 10.25%, results in the MVA. The MVA is the present value of the excess profits to be earned by the corporation in all future periods by both old and new assets alike. To understand this structure, it may be helpful to recall that the figure of \$0.959m. was previously shown to be equal to the value of a perpetuity with an annual value of \$0.057535m. (the annual surplus profit to be generated by the investment of \$6m. at the end of period one), using the growth rate (6%) as the discount factor: it is the aggregate surplus profit to be generated in period one by all prior capital investments.

This leaves the top sliver which lies between the asterisks and the squares. This is the present value, year by year, of the market rate of return on the assets in situ at the beginning of period one. Its period one value is the figure of $7.109m. derived in Table 1-4, discounted at the ECC. This amount is discounted through infinity to give the capital sum of $69.359m. shown in Table L1-1 as that portion of the book cost of the corporation's capital employed that has been financed from equity sources – retained earnings - over its infinite past.

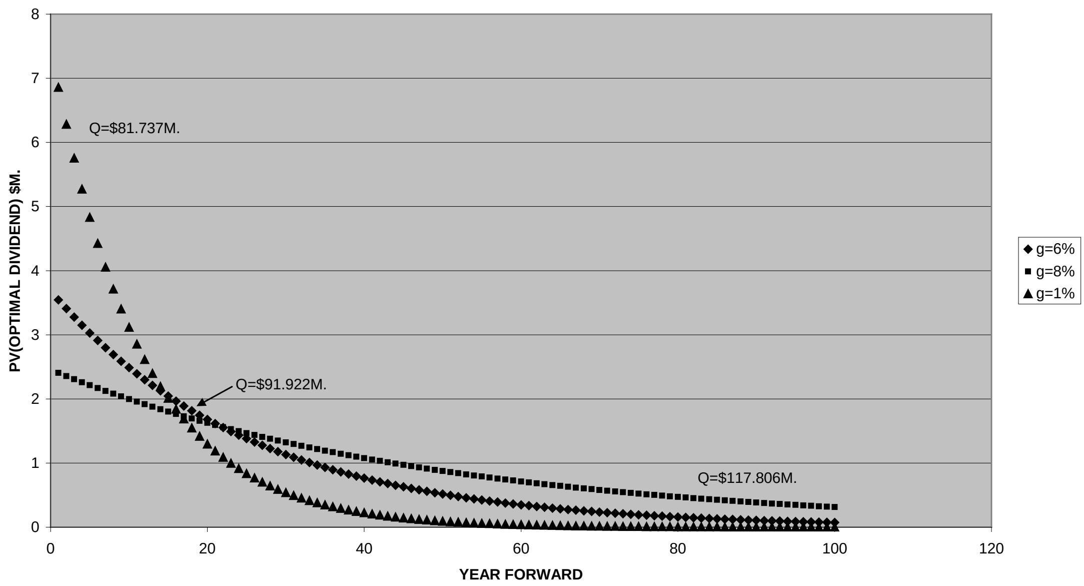
It is unlikely that any investor would think it appropriate to realise the value of her investment in the corporation according to the pattern indicated by the dividends (the diamonds) in Figure L1-4 if, as recommended, the corporation distributes its residual net cash flow by operating a stock repurchase programme. It is then even less likely that any investor would think it sensible to attempt to realise her investment in the corporation according to the pattern indicated by the PVGO model, but it may be noted that the duration of the quasi cash flow in Figure I1-5 works out at about 14 years: this compares with the figure of 26 years stated earlier as the duration of the optimal dividend (repurchase) flow. As already noted, if the squares were to be viewed as a curve trending downwards through to infinity, the area under this curve would be equal to the area under the curve given by the diamonds. This equality contrasts with the marked divergence in the crude cash flows shown earlier in Figure L1-4.

Growth and the Pattern of Dividends

Figure L1-5 above demonstrates various curves of present values relating to the case of 6% growth in Table L1-1 earlier. The present section and that following now set out comparisons between the case of 6% growth, and the cases of 1% and 8% growth also shown in the table. The three corporations depicted are similar in that they each have the same capital employed - $100m. at the beginning of period one - and they are equally profitable at the level of the NOPAT – ROCE of 10.054%. OPP1 showed that, in order to maximise MVA when the corporations face the same cost of capital functions, they will use the identical WACC in determining their capital expenditure levels period by period. Then, by positioning their MEC curves appropriately, in particular, assuming the same value for the intercept of the MEC in each case, it was possible to establish a common level of profitability. Of course, different rates of shift were assumed for the corporation's MECs: in this way, each was set up to have the same period one capital employed. But there the similarities end. A higher growth rate sets up a series of positive effects that mean that faster growing corporations may manifest significantly higher enterprise values and, more importantly, higher equity market capitalisations. More importantly, MVA expands dramatically with the rate of growth.

Figure L1-6 below shows the pattern of the present values year by year over 100 years of the dividends for the cases of the corporations growing at 1%, 6% and 8% in Table L1-1.

FIGURE L1-6



The diamonds again relate to the 6% growth case already exhibited in Figure L1-5 above: the period one value of the series is the payout figure of $3.907m. discounted for one period at 10.25%. As noted previously, the discounted dividends (or preferably, repurchase amounts) have a present value of $91.922m. as indicated on the graph, and a duration of about 26 years.

The corporation growing at only 1% per annum – now the triangles – initially pays higher dividends, but these discounted values fall away relatively quickly. The period one payout amounts to $7.581m. before discounting: this figure may be found as before in Table L1-1. As may be seen in the table, the equity market capitalisation in this case amounts to $81.737m., and the duration is substantially lower at 12 or so years.

The squares then relate to the case of 8% growth. Market capitalisation rises to $117.806m., based on a period one dividend of only $2.651m., and duration stretches out to about 49 years. The lower period one payout reflects two effects: firstly, the faster the corporation grows, the higher its debt ant thus its debt servicing costs, and secondly, the greater the amount retained to finance new investment. The points on the curves are, of course, asymptotic to zero, though in this case there remains a visible gap between the squares and this asymptote even after 100 years.

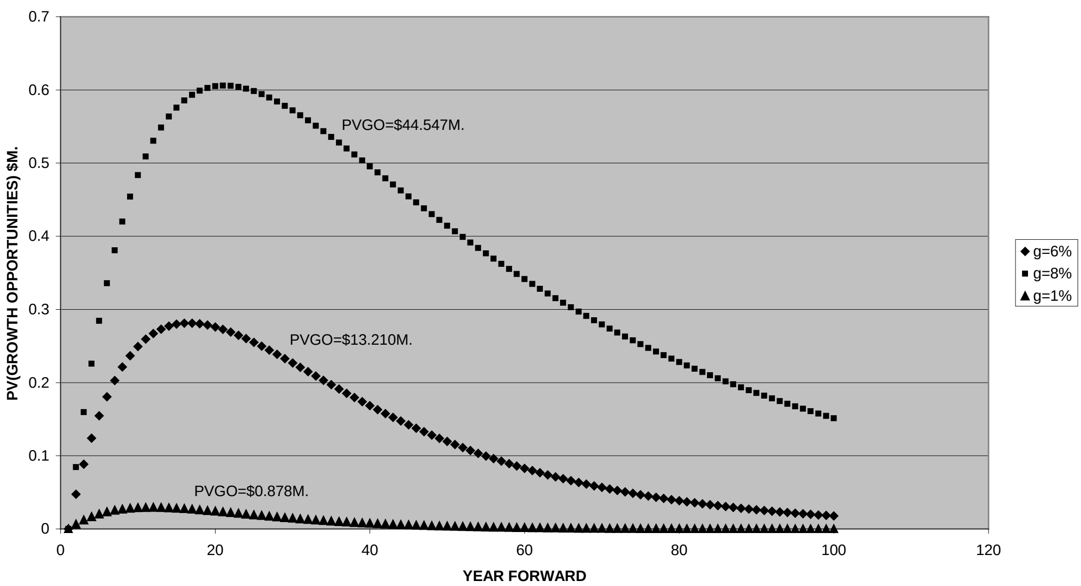
The Build-up of the PVGO over Time

A similar exercise to the above may be performed in relation to the components of the PVGO model. As before, the analysis builds up to the equity market capitalisation of the corporation by depicting firstly the PVGO itself, and then the MVA.

Figure L1-7 overleaf shows the build-up of the present value of the growth opportunities for corporations growing at 1%, 6% and 8%: the legend is the same as for Figure L1-6. The diamonds depict the same series for the case of 6% growth as the triangles in Figure L1-5 (thought he scales are different, of course).

As argued earlier, the PVGO takes the value of zero in period one, as the surplus profits on all previous capital expenditures, including that on the investment made at the end of period zero, are captured in the return on the period one assets. In the case of 6% growth, it was calculated that the investment of $6m. at the end of period one would generate in perpetuity a surplus profit of $0.057535m., commencing in period two. This figure may be derived from Table L1-1 as the product of the period one retained earnings - $4.162m. - and the difference between return on equity and the equity cost of capital – 1.383% (=11.633-10.250). As in Figure L1-6, the diamonds depict the case of 6% growth, and the point for period two is obtained by discounting the excess profit over two periods. In the case of 1% growth – the triangles - the period two excess profit amounts to $0.00831m. (=0.728*(0.11392-0.10250)): the corresponding figure for the case of 8% growth – the squares –

FIGURE L1-7



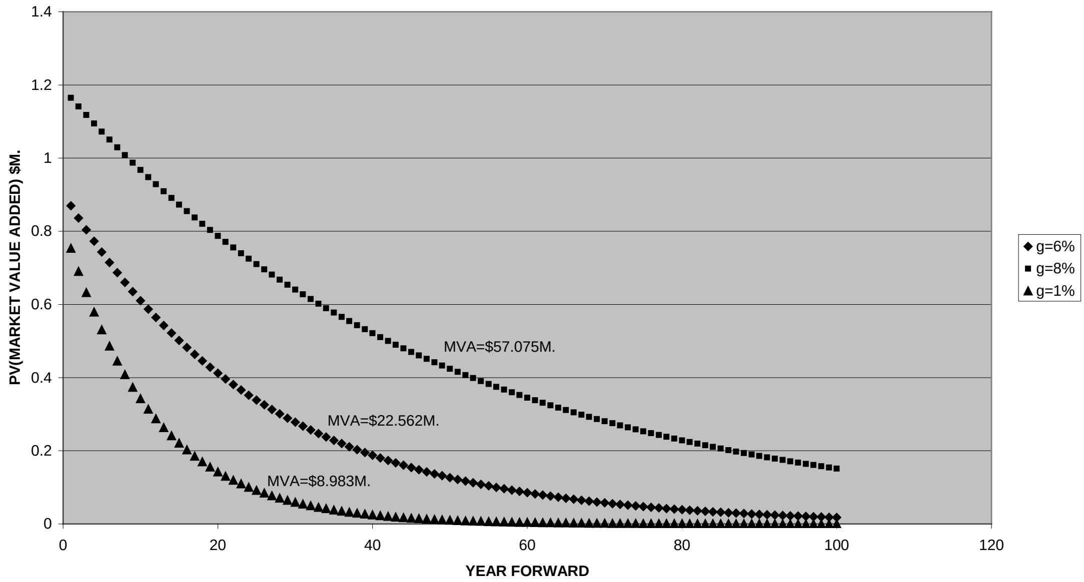
is $0.10277m. (=4.859*(0.12365-0.10250)). These amounts are similarly discounted over two periods to give the period two values of the triangles and the squares.

As may be seen in Table L1-2, the points sum to $0.878m., $13.210m. and $44.547m. respectively, so that PVGO makes the major contribution to driving the differences in the equity market valuations of the three cases. The yearly contribution to the present value is at a maximum in year 12 for the corporation growing at 1%: the maxima occur at 17 and 21 years for the other two series respectively.

Figure L1-8 overleaf shows the build-up of market value added for the three corporations. The diamonds again represent the case of 6% growth: this same series was previously depicted by the asterisks in Figure L1-5 (again there is a change of scale).

The MVA is the sum, period by period, of the surplus profits earned by the assets held at the beginning of period one, and the surplus profits earned by all subsequent investments up to that point in time. For the case of 6% growth, the surplus profit earned in period one by the initial capital employed may be extracted from the accounts in Table L1-1 by multiplying the initial balance sheet equity - $69.359m. - by the difference between the ROE and the ECC – 1.383% as calculated above. This

FIGURE L1-8

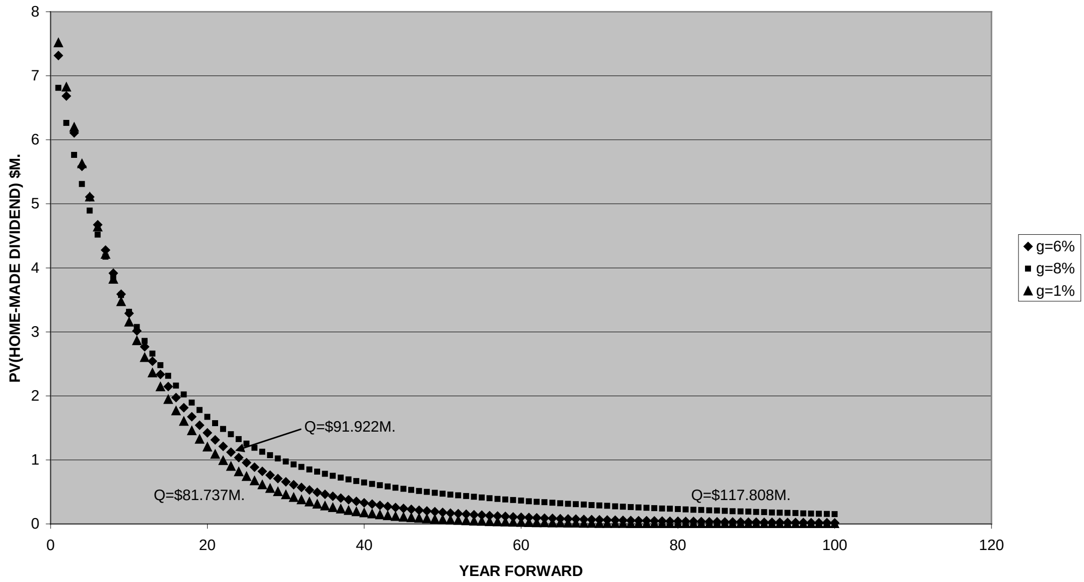


gives the figure of $0.959m., and the period one diamond represents this value discounted for one period at the ECC. To give the period two figure, it is necessary to add in the surplus profit on the period one investment - $0.057535: the second diamond is thus the figure of $1.01635 discounted over two periods. The MVA for the 6% case builds to a value of $22.562m., and this figure appears in both Tables L1-1 and L1-2. The values for the MVAs for the cases of 1% growth and 8% growth are also noted on the face of the graph.

Finally, Figure L1-9 overleaf shows the build-up of equity market capitalisation over time under the PVGO model. Here the diamonds represent the same series as the squares in Figure L1-5, the case of 6% growth. The squares sum to the equity market capitalisation of the corporation growing at 8%, while again the triangles depict the case of 1% growth.

The period one value of the series for the 6% growth case is simply the earnings figure shown in Table L1-1, $8.068m., discounted for one period. Before discounting, the figures for the 1% and 8% cases are respectively $8.288m. and $7.509m. The fastest growing corporation starts at a lower figure because the difference in the equity invested – a reduction of 16.53% compared to 1% growth - is greater in absolute terms than the corresponding difference in the ROE – and increase

FIGURE L1-9



of 8.54%. These effects follow from the greater use of debt, and the payment of higher interest charges, by the corporation growing at 8%. It takes eight years for the faster growing company to pull ahead, as may be seen in the figure: gradually the greater PVGO of the 8% case asserts itself. Again, the sums of the series, the market capitalisations of the three corporations, are shown on the face of the graph.

As noted earlier, the underlying series of the PVGO model do not represent meaningful accounting cash flows, but for the record, they have durations of about 11 years, 14 years and 29 years respectively for the cases of 1%, 6% and 8% growth. The corresponding durations for the dividend model were stated above to be 12 years, 26 years and 49 years.

Conclusion

Keynes (1936) built extensively on the concept of the marginal efficiency of capital to develop his investment demand schedule in the General Theory of Employment, Interest and Money. The present paper has looked behind this analysis to establish the demand for investment funds on the part of the individual corporation. By ranking, each period, its investment projects in descending order of internal rate of return, the corporation may draw up a schedule showing the dependence of the associated prospective annual NOPAT on the total amount invested. The gradient of this relationship (this is the derivative if the function is treated as being continuous) is called the marginal efficiency of capital expenditure, and it is the MEC (for instance, in Figure L1-2 above) which acts as the corporation's demand for investment funds.

The supply of funds is given by the corporation's WACC (a constant), and the intersection of demand and supply determines its optimal level of capital expenditure each period. The NOPAT applicable to this investment may then be seen as the area under the MEC up to this same level. The present paper has been concerned with the division of this NOPAT between that portion that fairly rewards the equity and debt invested at their factor costs, and that portion that may be interpreted as a surplus, sometimes called a 'quasi-rent' in economics. A lower rectangle set at the level of the BSWCC generates the income necessary to cover the factor costs through infinity: the residual upper quadrilateral then represents a surplus which accrues to the stockholders period by period. The present value of the corporation's growth opportunities (PVGO) is then the sum of the present values of the upper quadrilateral through infinity for the investment made in period one, plus the sums of the present values of similarly constructed quadrilaterals in respect of each of the investments the corporation may be expected to make in future years.

This framework in fact makes it possible to analyse the equity market capitalisation of the corporation into three components: the present value of the factor return to the corporation's current (beginning period one) balance sheet equity (part of the capital employed), the present value of the surplus profits to be earned in perpetuity by these initial assets, and the PVGO. The sum of the latter two components constitutes the corporation's market value added (MVA). MVA is the net present value of all the corporations investments, both those that have been laid down in the past, and those that remain to be created in the future. It was this magnitude which was identified as the corporation's maximand in OPP1, though this effectively amounts to maximising the PVGO. For the purpose of the present analysis, the value of the corporation's initial assets, and the income they generate, are taken as a datum: the model takes no account of the possibility that, in a dynamic economic environment, management time and effort will be expended on improving the operating efficiency and profitability of the firm as it stands at the beginning of period one.

This lemma has provided the occasion to review the Modigliani-Miller analysis demonstrating the irrelevance of the dividend decision. Although the construction of the MEC takes account of corporation tax, personal taxes – income tax and capital gains tax – have been ignored in this analysis. The most logical way for the corporation to react to personal taxes is for it to pay out its 'dividend cash' by buying in its own stock: in this way, investors subject to income tax on their cash dividends receive, as do all stockholders, an equivalent capital gain instead, which may also be deemed free of tax. Other things remaining the same, no stockholder loses as a result of this process, and some – those otherwise liable for income tax payments - will be better off. The corporation may indeed still over-distribute by buying in its own shares, just as it may by overpaying its cash dividend, but it will then need to raise new equity to part-finance its optimal capital expenditure programme. As buying and selling simultaneously a given number of shares at a common price has no economic significance (ignoring transaction costs, of course), any such sale of new shares may be netted-off against the repurchase plan in the cash flow statement, so that the optimal payout amount - and thus the payout ratio also – remain present in the accounts, albeit that presence is only implicit. The so-called 'Dividend Irrelevancy Proposition' therefore understates the importance of the constant growth case, where the corporation remains self-financing with respect to equity by retaining a fixed proportion of its earnings for reinvestment each period. As demonstrated in OPP1, the resulting optimal payout ratio will generally be a falling function of the corporation's growth rate.

As the corporation's net profits (earnings) will also be a declining function of the growth rate (due to rising debt and interest payments), a faster growing company will generate only a severely reduced immediate distributable amount relative to its more pedestrian counterparts (assuming, as before, a common period one asset base, and a common NOPAT). However, any such shortfall will be compensated for by the enhanced present values of the growth opportunities of the faster growing entity, and the factors identified in the Leibowitz and Kogelman formula (Figure L1-3 above) demonstrate how this effect comes about. The growth factor is self-evidently a positive function of the growth rate. It is less obvious that the franchise factor will also increase with the corporation's growth rate. The franchise factor is the difference between the inverse of the equity cost of capital and the inverse of the return on equity. Its rising tendency reflects the way the ROE increases with growth as an increasing reliance on debt reduces the proportion of equity in the balance sheet: additional debt financing is further encouraged by the tax shield effect. While the impact of those higher interest payments on the earnings of the faster growing corporation is mildly depressing, this is more than offset by multiplying earnings successively by the growth factor and the franchise factor to give a substantially enhanced PVGO. The corporation's equity market capitalisation is then the sum of the PVGO and the present value of a perpetuity equal to the earnings generated by the initial capital employed (equation 2 above). It is in this way that the low dividend yield of the faster growing corporation becomes sustainable.

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